# General announcements

# The Island Series:

You have been kidnapped by a crazed physics nerd and left on an island with twenty-four hours to solve the following problem. Solve the problem and you get to leave. Don't solve the problem and you don't.

**The problem**: You are told you will be given 5 seconds to stop two different object using a constant force of your choice (it doesn't have to be the same force for each object). Before you see either object, though, you must say which will take the greatest force to stop.

*You dissent* saying you don't have enough information to make the call, so you are given *two questions* (not "which force is bigger" or "which body experiences the largest acceleration"). From the responses to those two questions, you are to determine which body will require the larger force.

What are your questions?

# *What You Know To Date*

*A moving object* has KE;

*Doing work* on an object can change its KE;

*Adding energy* increases KE and v while removing energy decreases KE and v.

*Work is done* when a force component along the line of motion is applied over a distance.

*To stop a body*, negative work must be done.

*The problem:* We are NOT dealing with a force exerted over a distance (i.e., an energy-related quantity), we are dealing with a force exerted over a period of time . . . so how do we deal with that?

# *Solution to Island Problem*

*What* governs *stopping force* requirements over time? The two parameters that will matter are:

> *The mass of each body* (the bigger the mass, the larger the force required to stop the body in a given amount of time); and
>
> *The body's velocity* (the faster the body is moving, the greater the force required to bring the body to a stop in a given amount of time);

# CHAPTER 9: Momentum

*When physicists* run into a qualitative question like the one posed in the Island Series question, they will often take the parameters that are key to understanding the solution to the problem and multiply them together to get an overall governing relationship (that is, after all, where the idea of *work* came from). The idea is that if that quantity is large, the phenomenon being examined will be pronounced, and if small, not so much.

*In this case,* the product of the mass and velocity produces a vector

$$\vec{p} = m\vec{v}$$

called *MOMENTUM*.

*Kindly notice* that this relationship is really *three* equations in one— it denotes momentum in the x-direction, in the y-direction and in the z-direction.

*Interestingly, Newton* didn't originally write his *second law* as $\vec{F}_{net} = m\vec{a}$, he wrote it as:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

*Taking that derivative* yields:

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt}$$

$$= m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

*The first part* relates *force* to the *acceleration of the object*. It just equals $m\vec{a}$. The second part is related to how *force is required* to deal with situations in which the *mass* of a moving object *changes*. An example of such situations might be a rocket whose mass is changing as it burns fuel upon lift-off, or possibly a dump truck that is being loaded with gravel as it moves. As problems like that are not generally addressed in classes like this, we end up with Newton's Second Law looking like:

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

*What is useful* is that if we focus on just one direction, say, the x-direction, we can use the idea of *momentum* in conjunction with *Newton's Second Law* to write a relationship that links force and changing momentum to a single body's motion. That is:

*Over a differentially* small time interval dt:

$$F_x = \frac{dp_x}{dt}$$
$$\Rightarrow \quad F_x dt = dp_x$$

*Over a macroscopically* large time interval $\Delta t$:

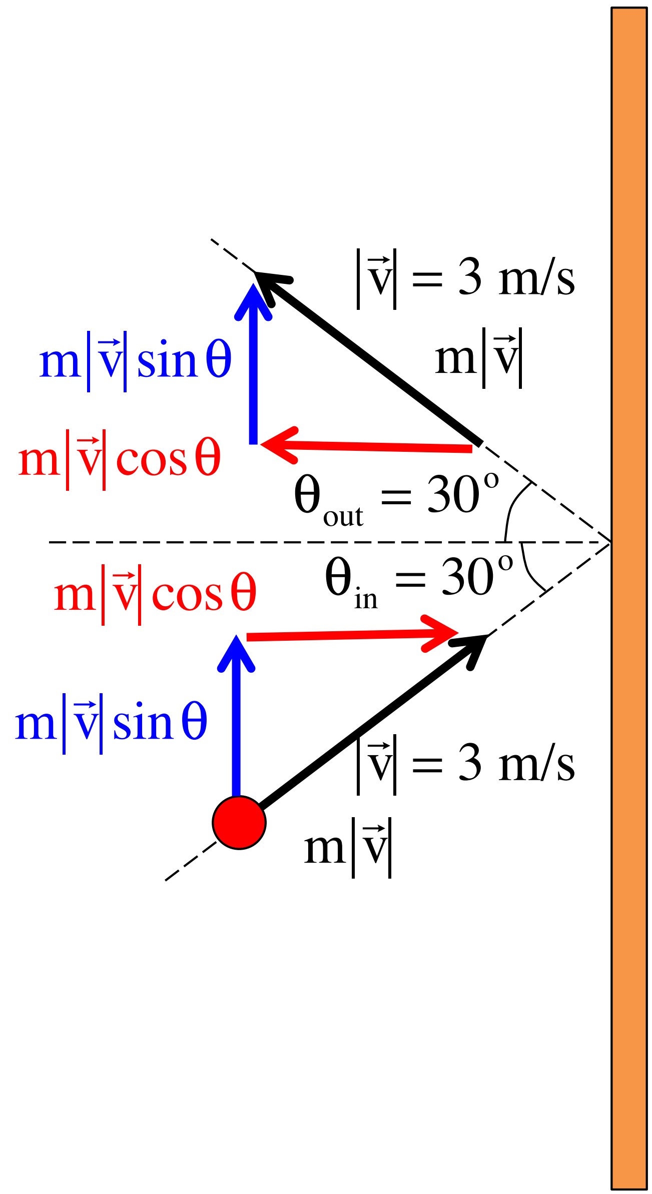
$$F_x = \frac{\Delta p_x}{\Delta t}$$
$$\Rightarrow \quad F_x \Delta t = \Delta p_x$$

--*The* $F_x \Delta t$ *quantity* is called the *IMPULSE* on the body.

--*The* $F_x \Delta t = \Delta p_x$ *quantity* is called the *impulse relationship.*

*Consider:* You are looking down on a 2 kg puck sliding over a frictionless surface moving at 3 m/s as shown. It bounces off a wall as shown. What is the net impulse on the puck?

*The key is* to realize that you have to treat the momentum change *as a vector* (look at the momentum in the y-direction—it *isn't* changing, so you'd better not end up with math that suggests that it should . . . )

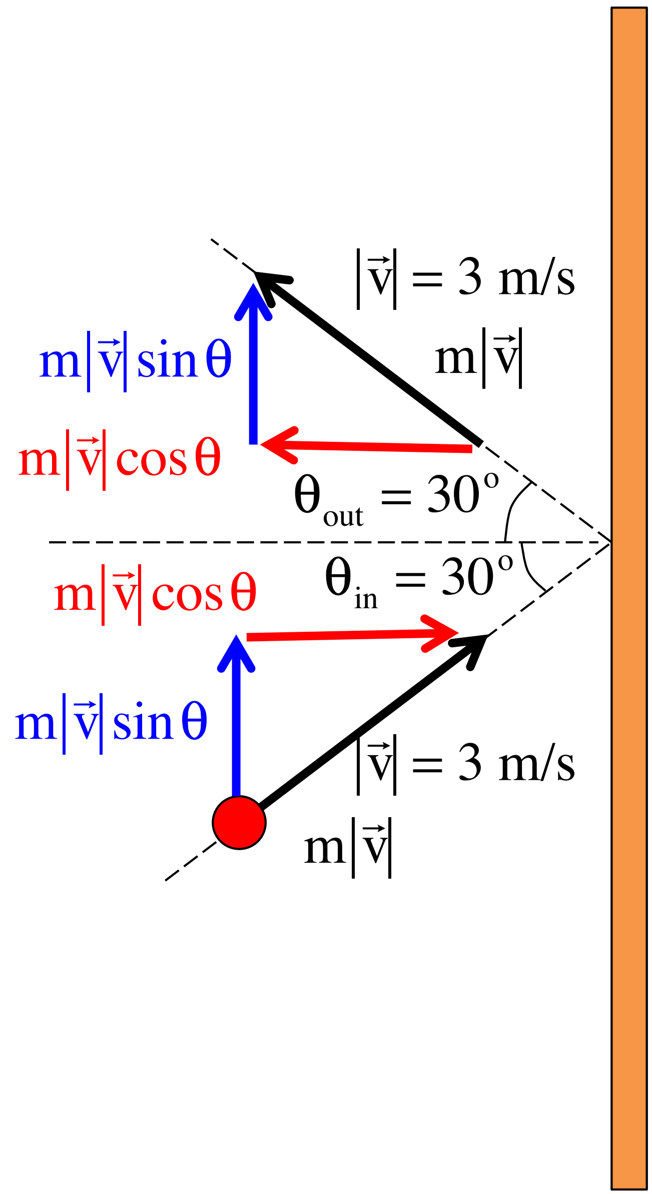



*In the x-direction:*

$$
\begin{aligned}
F_x\Delta t &= \Delta p_x \\
&= p_{x,2} - p_{x,1} \\
&= \left(-m|\vec{v}|\cos\theta_{out}\right) - \left(m|\vec{v}|\cos\theta_{in}\right) \\
&= -2m|\vec{v}|\cos\theta_{out} \\
&= -2(2\text{ kg})(3\text{ m/s})\cos\left(30^\circ\right) \\
&= -10.4\text{ kg}\bullet\text{m/s}
\end{aligned}
$$

*In the y-direction:*

$$\begin{aligned} F_y \Delta t &= \Delta p_y \\ &= p_{y,2} - p_{y,1} \\ &= \left(m|\vec{v}|\sin\theta_{out}\right) - \left(m|\vec{v}|\sin\theta_{in}\right) \\ &= 0 \end{aligned}$$




*So the net* impulse (which is normally characterized as a *J*, though the book uses *I* for reasons that are unclear):

$$\begin{aligned} \vec{J} &= \left(F_x \Delta t\right)\hat{i} + \left(F_y \Delta t\right)\hat{j} \\ &= \left(-10.4\hat{i} + 0\hat{j}\right) \text{kg} \bullet \text{m/s} \end{aligned}$$

*This makes perfect* sense as you would expect the impulse that would change the puck's motion to be away from the wall in the *minus x-direction*.

*This also* tells you something about our world:

*Want to keep* a driver safe during a car crash, pad the dashboard or, better yet, put air bags into the car. Why? Because when the driver goes from *60 miles per hour* to *zero miles per hour* due to a crash, the impulse (the *change of momentum*) will be what it will be, but the *time of impact* can be controlled (you want it to be as long as possible so the FORCE of impact is as small as possible. That is:

$$\left(F_{\text{on driver}}\right)\left(\Delta t_{\text{of crash}}\right)=\left(\Delta p_{\text{of driver}}\right)$$

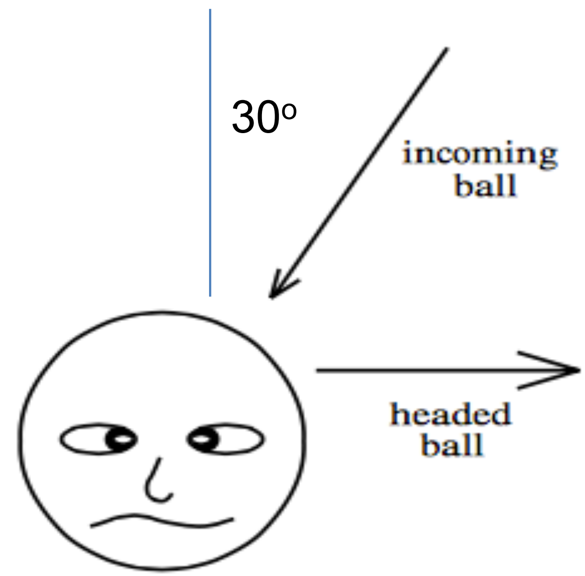
# *More on why impulse and momentum matter*

You are standing on a second-story balcony. Below you is a pool full of water, surrounded by concrete. If you had to jump, where would you want to land and why?

- Pool (duh). It's "softer" – meaning, it brings you to rest more gently than the concrete, which stops you pretty much immediately.
- This is all part of the impulse/delta-p relationship!
  - Either way, you'll have the same momentum on impact (same mass, fall same height so same velocity)
  - That momentum has to "go away" due to the impulse from the ground. The **impulse** to bring you to a stop is the same no matter which surface you hit.
  - This means $F_{concrete}t_{concrete} = F_{pool}t_{pool}$. In other words: short time to stop, bigger force. Longer time to stop, smaller force.
  - How do we use this principle in everyday life?

# Here's another one!

**7.30)** A .5 kg soccer ball comes down at a $30^o$ angle relative to the vertical moving at 25 m/s (this is the same as having a ball freefall approximately 100 feet). It is headed by a player, leaving the player's head at an angle of $90^o$ relative to the vertical moving with a velocity magnitude of 18 m/s.




**a.)** What was the ball's *momentum-change* off the player's head?

**b.)** If the collision takes .08 seconds, what is the *force* applied to the player's head (note that this is opposite the force applied to the ball)?

# Problem 6.3

- A tennis player claims they can serve a 0.145 kg tennis ball with as much momentum as a 3.0 g bullet moving at 1500 m/s.
  - a) How fast must the tennis ball be moving for this to be true?
  - b) Does the tennis ball or the bullet have more kinetic energy?

# *Center of Mass*

- So far, we've done all these problems assuming objects are "uniform spheres" – we ignore shape and mass distribution and treat them all as isolated points of mass $m$.

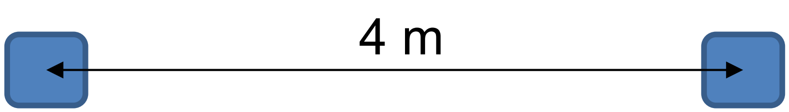


- In reality, objects are more complex! Thus, we use the **center of mass** to help approximate motion as a point for complex objects/motions.
  - The **<u>center of mass</u>** for an object or system is the point of equal mass distribution.
  - Where is the center of mass for these objects, roughly?

# Center of Mass

- Like most things in physics, the center of mass is relative to a coordinate system. You get to decide where your axes are, so pick places that make your life easier!

- Easy example: two 5-kg objects are sitting motionless 4 meters apart. Where is the center of mass of the system?




Logic tells us, since the objects are of equal mass, the center of mass of the system would be exactly halfway between them, right?

If x = 0 is at the center, then the center of mass is at x = 0 m.

If x = 0 is where the left-hand mass is, the center of mass is at +2 m.

If x = 0 is where the right-hand mass is, the center of mass is at -2 m.

All three of those put it at the same location! Just relative to different x = 0.

# Calculating center of mass

- To calculate the center of mass of a system when it's NOT so obvious, we need a formula.
- Thinking back to the easy example, we basically took the positions and masses of each object and "averaged" them out. In other words:

Mass times x-coordinate for each object, added up

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

X coordinate of the center of mass

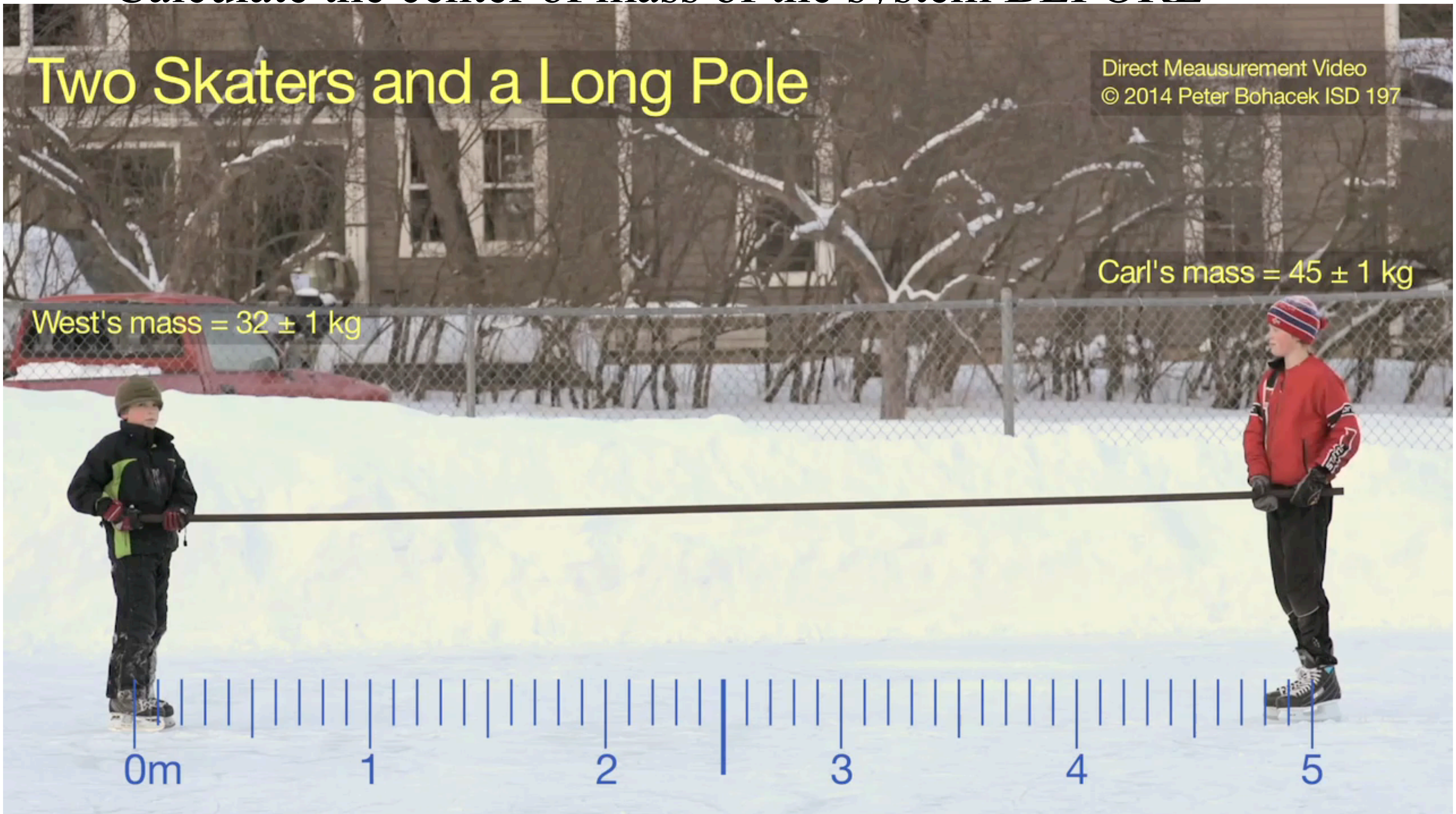
Sum of all the mass in the system

- We can do this in any direction (x, y, z), and in general form, it looks like this:

$$x_{cm} = \frac{\sum_{i=1}^{N} m_i x_i}{M} \qquad y_{cm} = \frac{\sum_{i=1}^{N} m_i y_i}{M}$$

# *Video example!*

- Calculate the center of mass of the system BEFORE



# *Example*

- There is a meter stick at the front of the class with masses taped at two different points on the meter stick. Determine the center of mass for the object.
  - Remember to define your x = 0 point clearly!